Midterm #1

MATH 332 — Differential Equations II
Monday, Feb 23, 2026

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 33 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (8 points) Let $M = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$.

(a) Compute e^{Mt}.

(b) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Problem 2. (2 points) If $M^n = \begin{bmatrix} 2-4^n & -2+2\cdot 4^n \\ 1-4^n & -1+2\cdot 4^n \end{bmatrix}$, then $e^{Mx} =$ $\boxed{}$.

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Problem 3. (6 points) Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 6a_n$ and $a_0 = 5$, $a_1 = -5$.

(a) Find an explicit (Binet-like) formula for a_n.

(b) Determine $\lim_{n\to\infty} \frac{a_{n+1}}{a_n}$.

Problem 4. (6 points) Fill in the blanks. None of the problems should require any computation!

(a) Consider a homogeneous linear differential equation with constant real coefficients which has order 5. Suppose $y(x) = 7 + 2x e^{3x} \cos(4x)$ is a solution. Write down the general solution.

(b) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + 5y = 4x^2 e^x$. Find a homogeneous linear differential equation which y_p solves.

You can use the operator D to write the DE. No need to simplify, any form is acceptable.

(c) Determine a (homogeneous linear) recurrence equation satisfied by $a_n = 4 - (2n+1)3^n$.

You can use the operator N to write the recurrence. No need to simplify, any form is acceptable.

Problem 5. (8 points) Determine all equilibrium points of the system $\frac{dx}{dt} = (x-2)y$, $\frac{dy}{dt} = y - x$. Classify the stability and type of each equilibrium point.

Problem 6. (3 points) Consider the following system of initial value problems:

$$\begin{array}{l} y_1'' + 2y_1 = 5y_2 \\ y_2'' + 4y_2 = 7y_1' \end{array} \qquad y_1(0) = 1,\ y_1'(0) = -2,\ y_2(0) = 3,\ y_2'(0) = 0$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{c}$.

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(extra scratch paper)